ACTivATE Exercise Documentation

Course in Which the Critical Thinking Exercise is Embedded

Course Title
Trigonometry

Instructor
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Applied Critical Thinking Exercise Overview

Exercise Title
Core-Card Reason Sheets

Description
Students develop a mathematical confidence using Core-Cards and take those skills to analyze complex problems on Reason Sheets. The students use critical reasoning skills to solve problems with multiple solutions and argue why their solutions work.

Discipline(s)/Subject Area
Mathematics

Grade or Course Level
High School Pre-calculus, Trigonometry

Time Needed
Two, 45-minute class periods, or for a few minutes every day

Foundation

High School Common Core Math Practices:

1. **CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.

2. **CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to _decontextualize_—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own,

3. **CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.

California Career and Technical Education Standards:

ED.D.D5.1 Understand the steps in the design process.
ED.D.D5.2 Determine what information and principles are relevant to a problem and its analysis
ED.FS.1.0 Academics
ED.FS.5.1 Apply appropriate problem-solving strategies and critical thinking skills to work related issues and tasks

Common Core Math Standards

M.6.NS.1.1 Compare and order positive and negative fractions, decimals, and mixed numbers
M.6.NS.1.2 Interpret and use ratios in different contexts
M.6.SDAP.3.1 Represent all possible outcomes for compound events in an organized way
M.8-12.G.3.0 Students construct and judge the validity of a logical argument

Student Objectives:

Upon completion of this unit, students will:
Have a solid foundation in essential math skills (fractions, decimals, percents)
Diagnose a problem. Design, produce, and test a solution to a problem
Understand that critical thinking and design is an iterative process
Understands that creativity is an important characteristic to apply to critical thinking.
Believe in his/her ability to design a solution to a problem and prove why it is a feasible solution.

Framing Concepts:
Students start by gaining a mathematical fluency by using numbers for a few minutes every day. They then must be able to apply that learning to a critical thinking problem. The student tries to answer problems with multiple solutions in multiple directions, once they have a possible solution, they present it in to the class and are able to defend their solution.

Assessment/Student Products

Pre-exercise Assessment
Fractional Reason Card #1
Algebra Reason Card #1
Trigonometry Reason Card #1

Post-exercise Assessment
Fractional Reason Card #3
Trigonometry Reason Card #5
Prove why the Reason sheet that does not work, does not work
Create your own reason card, final assessment

Materials & Resources
Core-Cards, Online videos, attached reason sheets

See video overview of how the sheets work in any math class from Pre-Algebra through Higher level math:
http://www.youtube.com/watch?v=Z1OZ4MliyIo&list=PLBn5gVyjYrPMCHzi_LeB9pk1M6q8n3Ygd&index=3

See video of Teacher using Core-cards and reason sheets in a Trig Class:
https://www.youtube.com/watch?v=a01wt3vnGYo

Exercise Details

Prerequisite Skills for Student Success
Students have:
- Foundational math skills, fractions, decimals, percents, proportions
- Trigonometry identities

Potential Student Misconceptions/Misunderstandings or Challenges with the Exercise
1. Students often assume there is only one solution, not multiple solutions.
2. Students expect each reason sheet to have a solution, sometimes there is not one and that needs to be proven in a logical argument.

Exercise
1. INTRODUCTION

   Core-Cards reinforce essential math skills while teaching creativity. The students are continuously practicing essential math skills: fractions, decimals, percents, while working with these cards. First lesson is to show the students the cards and have them use them as manipulative to solve the Reason Sheet. see video.
   Next is to argue why a certain card cannot work.

2. INSTRUCTION

   Prove that your solution works in all possible ways in writing and in presentation.
   Prove that a certain card cannot work by constructing a viable argument.
   Design and build your own Reason Card.
   Present a solution to the class explaining why it works in all possible ways.

3. CLOSURE

   The student presents a solution to the reason sheet to the class. They are able to defend their solution while the class questions the solution.

Instructor Comments/Reflection

Students will solve complex problems with multiple solutions, proving that their solution meets all the constraints of the multiple equations in the Reason Sheet.

Critical Thinking Skills:
4. Identify additional information needed to evaluate a hypnosis
9. Provide relevant alternative information when solving a problem
12. Use basic math skills when solving a problem
13. Identify suitable solutions for a problem using relevant information
17. Interpret viability of conclusions

Student Assessment

The assessment is a practical demonstration of a reason card filled in with corresponding written proof it works or why it does not work.

Student Assessment Rubric

The Student Assessment Rubric content is based on the Exercise's content and relevant evaluation criteria. Work with students to develop the criteria for each level of the rubric – Unsatisfactory through Above & Beyond.

Assignment	Unsatisfactory	Bare Minimum	Satisfactory	Above & Beyond
Filling in Reason Card	Student makes little/no attempt to identify possible solutions.	Student makes some attempt to identify solutions, equations work in one direction but not all directions.	Student accurately identifies the solutions, they work in all directions.	Student demonstrates a sophisticated level of understanding and finds multiple solutions that all work in all directions.
Written paragraph of reason card that works	Student makes little/no attempt to identify possible solutions.	Student makes some attempt to explain their solution, sentences are incomplete.	Student selects a correct solution and writes a viable argument why the solutions works in all directions.	Student selects a correct solution and writes a viable argument why the solutions works in all directions. Also creates a viable argument of the limits of the problem, by writing out domains of the equations.
Written paragraph of reason card that does not work	Student makes little/no attempt to identify possible solutions.	Student makes some attempt to explain their solution, sentences are incomplete.	Student clearly explains why the problem cannot work, using examples and explaining the possible values the variable could be.	Student writes a viable argument that demonstrate a sophisticated level of understanding, analysis, inquiry, and engagement. Clearly explains why no solutions can work because of the domain of the problem.
Create own reason card	Student makes little/no attempt to identify possible solutions.	Student creates a card that works in a few directions but does not check that it works in all directions.	Student creates a card that works in all directions and shows solutions to all problems.	Student creates a card that works in all directions and shows solutions to all problems. Student also identifies skills assessed by the person using the card.

	+		=	$\dfrac{1}{2}$
+		**+**		
	+		**=**	$\dfrac{3}{4}$
=		**=**		
$\dfrac{3}{8}$		$\dfrac{7}{8}$		

	+		=	$\frac{3}{8}$
+		+		
	+		=	$1\frac{3}{4}$
=		=		
$1\frac{1}{8}$		1		

	X		=	$\frac{15}{64}$
+		**X**		
	+		**=**	$1\frac{3}{8}$
=		**=**		
$1\frac{1}{8}$		$\frac{21}{64}$		

x	**+**	x	**=**	$\frac{1}{2}x$
+		**X**		
x	**+**	x	**=**	$1\frac{1}{8}x$
=		**=**		
$1x$		$\frac{1}{16}x^2$		

	+		=	
+	X	X		
	+		=	
=		=	=	

	X		=	$1\frac{7}{8}$
+		X		
	+		=	$1\frac{3}{8}$
=		=		
$1\frac{1}{8}$		$2\frac{5}{8}$		

	+		=	**1**
-	X	X		
	+		=	2
=		=	=	
$-cos^2x$		cos^2x		sin^2x

	X		**=**	$cot^2x\ cos^2x$
÷		**+**		
	-		**=**	cos^2x
=		**=**	**=**	
cot^2x		csc^2x		

	+		=	$2sec^2x - 1$
X		**-**		
	X		=	cos^2x
=		**=**		
csc^2x		$tan^2x \, sin^2x$		